**CC Algebra II**

**Test #3 – Quadratic Equations and Complex Numbers - Review**

**Formulas**

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<td>$20x^2 - 5x = 5x(4x - 1)$</td>
<td>Difference of Two Cubes</td>
<td>$27x^3 - 8 = (3x - 2)(9x^2 + 6x + 4)$</td>
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<td>Factor $x^2 + 4x - 12 = (x + 6)(x - 2)$</td>
<td>$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$</td>
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<td>Difference of Two Squares</td>
<td>$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$</td>
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<td>$x^2 - 49 = (x - 7)(x + 7)$</td>
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**Solving Quadratic Equations**

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<td>By Factoring - Write the equation in factored form and solve using the Zero-Product Property</td>
<td>By Completing the Square - Isolate $x^2 + bx$, then add $(\frac{b}{2})^2$ term to make a perfect square trinomial</td>
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Using the Quadratic Formula - Write the equation in standard from, then plug into the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Complex Numbers**

| The imaginary unit $i$ is defined as $i = \sqrt{-1}$ | Note that $i^2 = -1$ | Standard Form is $a + bi$ |

**Solving Quadratic Equations**

- If $b^2 - 4ac > 0$, Then the equation has two real solutions
- If $b^2 - 4ac = 0$, Then the equation has one real solution
- If $b^2 - 4ac < 0$, Then the equation has two imaginary solutions
**Regents Problems**

**August 2017 Regents Exam**

1. Which expression is equivalent to \((3k - 2i)^2\), where \(i\) is the imaginary unit?
   a. \(9k^2 - 4\)
   b. \(9k^2 + 4\)
   c. \(9k^2 - 12ki - 4\)
   d. \(9k^2 - 12ki + 4\)

2. The roots of the equation \(x^2 + 2x + 5 = 0\) are
   a. -3 and 1
   b. -1, only
   c. \(-1 + 2i\) and \(-1 - 2i\)
   d. \(-1 + 4i\) and \(-1 - 4i\)

3. What are the zeros of \(P(m) = (m^2 - 4)(m^2 + 1)\)?
   a. 2, and -2, only
   b. 2, -2, and -4
   c. -4, i, and -i
   d. 2, -2, i, and -i

**June 2017 Regents Exam**

4. The expression \(6xi^3(-4xi + 5)\) is equivalent to
   a. \(2x - 5i\)
   b. \(-24x^2 - 30xi\)
   c. \(-24x^2 + 30x - i\)
   d. \(26x - 24x^2i - 5i\)

5. The solution to the equation \(4x^2 + 98 = 0\) is
   a. \(\pm 7\)
   b. \(\pm 7i\)
   c. \(\pm \frac{7\sqrt{2}}{2}\)
   d. \(\pm \frac{7\sqrt{2}}{2}\)

6. Over the set of integers, factor the expression \(4x^3 - x^2 + 16x - 4\) completely.

**January 2017 Regents Exam**

7. Factored completely \(m^5 + m^3 - 6m\) is equivalent to
   a. \((m + 3)(m - 2)\)
   b. \((m^2 + 3m)(m^2 - 2)\)
   c. \(m(m^4 + m^2 - 6)\)
   d. \(m(m^2 + 3)(m^2 - 2)\)
8. The solution to the equation \(18x^2 - 24x + 87 = 0\) is
   a. \(-\frac{3}{2} \pm 6i\sqrt{158}\)
   b. \(-\frac{3}{2} \pm \frac{1}{6}i\sqrt{158}\)
   c. \(\frac{3}{2} \pm 6i\sqrt{158}\)
   d. \(\frac{3}{2} \pm \frac{1}{6}i\sqrt{158}\)

9. Express \((1 - i)^3\) in \(a + bi\) form.

August 2016 Regents Exam

10. Which equation has \(1 - i\) as a solution
   a. \(x^2 + 2x - 2 = 0\)
   b. \(x^2 + 2x + 2 = 0\)
   c. \(x^2 - 2x - 2 = 0\)
   d. \(x^2 - 2x + 2 = 0\)

11. The completely factored form of \(2d^4 + 6d^3 - 18d^2 - 54d\) is
   a. \(2d(d^2 - 9)(d + 3)\)
   b. \(2d(d^2 + 9)(d + 3)\)
   c. \(2d(d + 3)^2(d - 3)\)
   d. \(2d(d - 3)^2(d + 3)\)

12. Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?
   I \((m + p)^2 = m^2 + 2mp + p^2\)
   II \((x + y)^3 = x^3 + 3xy + y^3\)
   III \((a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2\)
   a. I, only
   b. I and II
   c. II and III
   d. I and III

13. Simplify \(xi(i - 7i)^2\), where \(i\) is the imaginary unit.
1. **ANS:** C \((3k - 2i)^2 = 9k^2 - 12ki + 4i^2 = 9k^2 - 12ki - 4\)

2. **ANS:** C 
   \[x^2 + 2x + 1 = -5 + 1\]
   \[(x + 1)^2 = -4\]
   \[x + 1 = \pm 2i\]
   \[x = -1 \pm 2i\]

3. **ANS:** D
   \[P(m) = (m^2 - 4)(m^2 + 1) = (m - 2)(m + 2)(m^2 + 1)\]
   \[m - 2 = 0; m = 2 \quad \text{and} \quad m + 2 = 0; m = -2 \quad \text{and} \]
   \[m^2 + 1 = 0 \quad \Rightarrow \quad m^2 = -1 \quad \Rightarrow \quad m = \pm \sqrt{-1} \quad \Rightarrow \quad m = \pm i\]

4. **ANS:** B
   \[6xi^3(-4xi + 5) = 24x^2i^4 + 30xi^3 = -24x^2(1) + 30x(-1) = -24x^2 - 30xi\]

5. **ANS:** D
   \[4x^2 = 98\]
   \[x^2 = \frac{98}{4}\]
   \[x^2 = \pm \sqrt{\frac{-49}{2}} = \pm \frac{7i}{2} = \pm \frac{\sqrt{5}}{2}\]

6. **ANS:**
   \[x^2(4x - 1) + 4(4x - 1) = (x^2 + 4)(4x - 1)\]

7. **ANS:** D
   \[m^5 + m^3 - 6m = m(m^4 + m^2 - 6) = m(m^2 + 3)(m^2 - 2)\]

8. **ANS:** D
   \[\frac{8\sqrt{-8} - 4(6x + 29)}{2(6)} = \frac{8\sqrt{-62}}{12} = \frac{8\sqrt{62}i}{12} = \frac{\sqrt{62}i}{3} \pm \frac{1}{6}\sqrt{158}\]

9. **ANS:**
   \[(1 - i)(1 - i)(1 - i) = (1 - 2i + i^2)(1 - i) = -2i(1 - i) = -2 + 2i = -2 - 2i\]

10. **ANS:** D

11. **ANS:** C
   \[2d(d^3 + 3d^2 - 9d - 27)\]
   \[2d(d^2 + 3) - 9(d + 3)\]
   \[2d(d^2 - 9)(d + 3)\]
   \[2d(d + 3)(d - 3)\]
   \[2d(d + 3)^2(d - 3)\]

12. **ANS:** D
   \[(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3\]

13. **ANS:**
   \[xi(-6i)^2 = xi(36i^2) = 36xi^3 = -36xi\]